Evaluation of Demand Risk Mitigation in PPP Projects

EIB University Research Sponsorship Programme

Draft 1

Updated: March 13, 2007
Draft
FOREWORD

This draft research was prepared by the Transport Research Center (TRANSYT) with the sponsorship of European Investment Bank.
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1.- STATE OF THE ART

Demand risk in PPP projects can be defined as the inability to determine the behaviour of real traffic movement compared to forecasted traffic. In this section we present the state of the art analysis of traffic risk behaviour and traffic risk mitigation in PPP projects.

1.1 The Traffic Risk Problem

According to the Green Book of the HM Treasury (2003), there is a demonstrated systematic tendency for bidders to be very optimistic when they participate in public work tenders. This situation affects key project parameters such as revenues, timing, costs (both capital and operational), and so on. This tendency often takes the form of increasing estimates of revenues, and decreasing estimates of costs.

The United Kingdom authorities commissioned Mott MacDonald (2002), an important consultant firm, to study public procurement. The research found that in the United Kingdom, during the last 20 years, different kinds of public procurement projects tended to be optimistic in its estimates of future revenues. Although the optimism bias was presented in traditional and PFI projects, traditionally procured project forecasts turned out to be less accurate than PFI project forecasts. This difference was attributed to the way British authorities grant PFI projects by negotiating with few preferred bidders. As PFI projects transfer the project risk from the public sector to the private sector, the level of diligence to establish the business case in PFI procurement turned out to be more accurate.

The study identified some common project risks. The document demonstrated that optimism bias for projects was not sector-specific, because similar levels of optimism bias were recorded for different project types across sectors. It is notable that the level of optimism bias recorded for a project was dependant on the project capabilities of the management team rather than on the number of risks associated with the project.

The study also provided a useful mechanism to measure the typical optimism bias depending on project types. Based on the results, the Mott MacDonald study provides some guidelines for planners in order to adjust demand and costs for projects at the procurement stage. The Mott McDonald study recommended that the way to reduce the levels of optimism bias is to actively promote knowledge transfer and knowledge sharing from completed and successful projects. They could take the form of external seminars, papers and methodological archiving of key documents inside the public agencies in charge of public work projects.
Another international study on toll road traffic forecasting performance explored the matter in 32 toll-road case studies. It was made by Bain and Wilkins (2002) for Standard & Poor’s. The study was updated two years later. The authors showed how optimism bias was present in toll road traffic forecasting. The study found that, on average, ultimate traffic volumes were about 70% of the predicted values. The main goal of the Standard & Poor’s study was to provide guidelines for toll-road credit investors. The results of the Standard & Poor’s study are particularly interesting for project finance lenders who have to evaluate the reliability of traffic forecasts prepared by the-promoters of the project.

The authors proposed a “Traffic Risk Index” to analyze traffic forecasts. This index is defined according to the characteristics of each toll road and the particular circumstances under which the forecasts have been prepared. The traffic risk index is estimated in terms of several variables that may affect traffic forecasts. These are: tolling regime, tariff escalation, forecast horizon, toll-facility details, survey and data collection, private/commercial users, micro-economic factors and traffic growth. The index is a useful checklist to assess the specific risks in infrastructure projects.

Although, the Standard & Poor’s study has an investment/credit analysis perspective, the study found two important features of traffic forecasting. First, toll-road projects with high levels of inherent uncertainty appear to be more susceptible to large forecasting errors. And second, the study showed that the stakeholder who has commissioned the traffic study is key to explain forecast errors. In this respect, there were significant differences between forecasts by banks, sponsors and bidders. Bank-commissioned forecasts consistently appear to be less prone to large errors than those commissioned by project sponsors and/or bidders.

Estache and Strong (2000) pointed out that project-finance investors on transport infrastructure have been aware of these issues only these past few years. Now, they are very careful with the risks associated with these kinds of projects. They say that, failures and mistakes in project finance deals in the 1990’s have forced them to design conservative financial structures, by carrying out comprehensive sensitivity analysis, taking into account the effects of macroeconomic factors, and the need to put proper incentives and sound institutional and regulatory arrangements.

Flyvbjerg et al (2005) showed that traffic forecast inaccuracy was common all around the world. To prove this, they used a large sample, covering 210 projects in 14 countries: Brazil, Chile, Denmark, Egypt, France, Germany, Hong Kong, India, Mexico, South Korea, Sweden, Tunisia, United Kingdom and the United States of America.

This study discovered important findings. First, traffic forecast errors do not have significant differences depending on the type of infrastructure (highways, bridges, and tunnels). Second, more than 50% of road projects have a difference larger than + 20% between actual and forecasted traffic for the first year. Third, underestimated traffic forecasts are closer to the predicted traffic than overestimated traffic forecasts.
According to this study, traffic forecast errors strongly depend on the personnel commissioned for this task. The study points out two causes for traffic forecasting errors: technical mistakes in the methodology, and the strategic behaviour of bidders. For the first case, Flyvbjerg recommends the use of a new forecasting method called “reference class forecasting\(^1\)”. This method consists in taking other “similar” projects as a reference to complement or validate the information about the project.

In the second case, Flyvbjerg et al. assumed that planners are not really concerned about obtaining accurate forecasts because their main goal is to promote competition in the PPP tender. Flyvbjerg proposed the use of two mechanisms to correct this issue: (i) public sector accountability through transparency and public control, and (ii) private sector accountability via competition and market control. One of his most important conclusions was “…it is highly risky to rely upon travel demand forecasts to plan and implement large transportation infrastructure investment\(^2\)”.  

Engel et al (2003) reviewed the experience in Latin America with highway privatisation during the last decade. They showed that in different countries, the traffic forecast inaccuracy problem was present as well. Although the main objective for privatising infrastructure facilities was to free up financial public resources, the ultimate PPP results were sometimes worse than expected because the funds from the initial savings were often diverted to subsequently bail out franchise holders. Engel et al. (2006) noted that traffic forecast inaccuracies are also the main cause of financial problems in different transport infrastructure projects in the United States.

### 1.2 Forecast Calculation and Traffic Inaccuracy

Wachs (1998) analysed the process of planning by considering planners and modellers (practitioners who make forecasts) as an important part of the problem of the inaccuracy of future traffic predictions. The analysis emphasizes the problems that planners often have to face to justify political decisions to built new transport infrastructure.

Niles & Nelson (2001) focused their study on the uncertainty within the models used to estimate traffic demand. Specifically, they indicated the different sources (variables) of forecasting bias like uncertainty in model design and structure, transportation network uncertainty, demographic and behaviour uncertainty, and uncertainty resulting from social/political bias. The study finished pointing out the dangers involved when governments blindly trust these forecasts.

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\(^1\) This method is very similar to that used to value comparable companies in economics.

\(^2\) Flyvbjerg et al (2005), pp. 140.
In the same line, Kriger (2005) focused his research on the way that models were used to predict traffic volumes. In order to improve the forecasts traffic, he suggested changing the classical four step process approach by adding new considerations and variables to old statistical models, or in some cases designing specific new models, according to the unique characteristics of new transport projects.

A similar study, conducted by Rand Europe (2005), analysed the uncertainty in the models used to make forecasts by public authorities in the Netherlands. They focused on the statistical error in the model and errors using parameter estimates. Their main conclusion was that the contribution of input uncertainty (e.g. in future incomes, car ownership levels, etc.) to these errors was generally much larger than that of model uncertainty (e.g. coefficients estimated with some error margin).

### 1.3 Traffic Risk and Renegotiation

Guasch et al. (2003) used a data set of around 1,000 privatizations projects in Latin America to assess the determinants of renegotiations in PPP projects. The study used data from transport, water and sewage projects granted during the 90’s in five countries (Argentina, Brazil, Chile, Colombia and Mexico). They built a panel including the contract characteristics of more than 300 contracts, together with information concerning the regulatory context, the type of price regulation in place, the institutional environment of the country, and the evolution of the economic environment.

The econometric analysis in this study led to a number of important conclusions concerning the determinants of firm-led renegotiations. The authors found that the conditions conducive to renegotiations were a combination of contract characteristics, regulatory environment and economic shocks. Specifically, renegotiations are more likely during recession or after devaluations, after elections, for concessions awarded before a proper regulatory agency was put in place, and regulation by a price cap. Furthermore, a solid institutional framework was shown to reduce the incidence of renegotiation, while a worse institutional environment, as captured by an index of bureaucratic quality, increases it.

In a subsequent study for the World Bank about concessions in Latin America, Guasch (2004) showed that between 1985 and 2000, the incidence of renegotiation was 55% of the total, in the case of transportation concessions. One important issue indicated by this study was how the regulatory regimen type conditioned who started the renegotiation. When the regulatory regimen was the “price cap” the initiators of renegotiations were predominantly (83%) the operators.

A different approach to renegotiation is provided by Ho (2004), who developed a model for analysing construction claims and opportunistic bidding in the procurement of public works. He presented a model based on game theory, which provides an
approach to analyse decisions and strategies in construction claims. He called his model the Claim Decision Model (CDM). He showed that when opportunistic bidding is encouraged by the government, renegotiation becomes the Nash equilibrium in a construction claim.

Following a similar methodology, Ho (2006) designed a new model for financial renegotiation in PPP projects. He developed a game theory based model for government rescue dynamics. He showed when and how the government will rescue a distressed project and what impact the government’s rescue behaviour has on project procurement and management.

1.4 Some Mechanisms to Mitigate Traffic Risk

The negative consequences of failing to manage traffic risk have spurred governments to develop and implement a wide range of traffic risk mitigation mechanisms. Across the spectrum of approaches, two common objectives are sought: first, increasing the completeness of concession contracts to reduce the potential for renegotiations; and second, establishing more equitable rules for sharing gains or loses between the concessionaire, the government and the users if renegotiation is required. Here we describe the three most common strategies to mitigate traffic risk in PPP projects: (1) “modification of the economic balance” of contracts; (2) traffic guarantee contracts; and (3) duration adjusted contracts.

The first approach, which was developed by France (Gomez-Ibañez and Meyer 1993) and recently adopted by Spain with some differences (Vassallo and Gallego 2005), consists of re-establishing the “economic balance” of the concession in case that the IRR falls below a minimum IRR stipulated in the contract. In some cases, a minimum IRR is accompanied by a maximum IRR. The upper bound limits the concessionaire’s profits if traffic is much higher than expected. Generally, the compensation measures to be adopted for re-establishing the economic balance of the contract are not pre-established but rather negotiated when the IRR falls above or below the target levels. Compensation can include change in toll levels, adjusting the contract length or the provision of public subsidies. Experience with this approach has revealed two major problems. Since the way to re-establish the economic balance of the contract is not fully specified, long and tedious renegotiations between the concessionaire and the government often occur. Also, the concessionaire has no incentive to reduce operating costs when the project IRR is close to the lower limit since falling below the limit allows a renegotiation of the contract. Overall, this mechanism has achieved the goal of increasing interest of the private sector in concession contracts; however it has been problematic because the agreements between the concessionaire and the government for re-negotiation may not yield net social gains.

The second approach –used in many countries such as Korea, Colombia, Chile, Dominican Republic, Malaysia and Spain– consists of guaranteeing either traffic or revenues. Failure to achieve the minimum level (specified by the public authority or the
bidder) triggers compensation from the government. In many contracts the lower limit is often complemented with an upper limit above which the revenues are shared between the government and the concessionaire. The main problem of the guarantee approach is that it cannot ignore the strong correlation between traffic and economic growth; the guarantee can have very negative consequences for the public budget if a recession occurs. Nevertheless, the mechanism has worked quite well in some countries such as Chile where, even during an economic recession, only 4 out of 29 transport concessions in operation at the end of 2004 performed below the minimum income guarantee band. This meant a subsidy from the government of only 6.24 US$ million compared to the 350 US$ millions invested (Vassallo and Sánchez Soliño 2006.) This mechanism has not worked so well in more unstable countries such as Colombia where traffic volume turned out to be lower than guaranteed levels for many concessions (Rufian 2002), which put a large strain on the government’s fiscal position.

The third approach, which has been adopted in several countries recently, is to match the duration of the concession to a predefined target, usually related to traffic or revenues. This approach was first applied in 1990 in the concession of the Second Severn Crossing in the United Kingdom. Although initially the government decided that the maximum period for the concession should be no longer than 30 years, the concessionaire -Severn River Crossing Plc.- proposed the basis of the length of the concession be set on a “Required Cumulative Real Revenue” (Foice 1998). This way, a total project revenue was established at 1989 prices, which, once collected from tolls income, would end the concession. Based upon traffic levels during the early years of the concession, it was expected that the concession duration would be ultimately 22 years, considerably less than initially predicted. Another similar concession was awarded in Portugal at the end of the 1990’s. The concession agreement was designed in order for the concession to expire no later than March 2028 or at a total cumulative traffic flow of 2,250 million vehicles (Lemos et al. 2004.) If the traffic is higher than expected the concession will finish earlier than 2028.

The best-known specification of this mechanism is called “Least Present Value of the Revenues (LPVR)” and has been extensively developed by Engel, Fischer and Galetovic (1997, 2001.) The authors of these papers working independently of the Severn Bridge experience developed this mechanism in response to a proposal from an official of the Ministry of Public Works of Chile (MOP). Under this mechanism, the concession is awarded to the bidder who offers the least present value of the accumulated revenues -discounted according to a discount rate pre-fixed in the contract. The concession ends when this LPVR is reached. Consequently, if the actual traffic is higher than expected, the concession will finish earlier whereas if it is lower, the concession will finish later. Traffic risk is thus shared among the concessionaire, the users (who pay tolls for a longer period if the traffic is ultimately lower than expected), and the government (who receives the concession later if traffic is lower.)

The LPVR mechanism was tried in Chile; however, only two concessions out of 29 were successfully awarded under this approach. Vassallo (2006) showed that this mechanism was less successful in practice than in theory was the strong opposition of the concession companies given the risk profile. Since the contracts established a maximum duration, the concessionaire bore the risk that the project would not reach the
LPVR requested before the end of the contract. Moreover, they saw no upside benefit because if the traffic were greater than expected the contract would end early.

Although LPVR has been mainly implemented in Chile, other countries have also adopted this technique. At the end of 2004, Portugal decided to apply LPVR to a new toll highway concession, the “Litoral Centro” highway. The concession will come to an end when the net present value (NPV) of the total revenue collected reaches € 784 million, subject to a minimum period of 22 years and a maximum period of 30 years. The so-called “third generation of concessions” carried out in Colombia implemented a similar mechanism based on awarding the contract to the bidder who requires the lowest accumulated revenues. Presently, the government of Colombia is studying the possibility of implementing LPVR in the next generation of concessions which are being defined now.

2.- APPROACH TO THE PROBLEM

The previous section gave some interesting features regarding traffic risk. First, the ultimate traffic during the contract life is essential for the profitability of PPP contracts. Second, inaccuracy of traffic forecasts is outstanding, especially in greenfield projects. Third, traffic risk is hardly manageable by both the private and the public sectors. Fourth, when traffic risk is fully transferred to the contractor, if traffic is much higher than expected the contractor may reap excessive profits, whereas if traffic is much lower than expected, costly renegotiations for the government may take place.

This is the reason why many governments all around the world are tending to implement measures to mitigate traffic risk. The prime two goals of these measures are to set up a more equitable traffic risk distribution and to reduce renegotiation during the life of the contract. Different mechanisms have been, therefore, put into effect such as minimum income guarantees, flexible term contract approaches and so on.

Up to now, only some parts of the problem have been reported in the literature. Flybjerg et al. (2005), and Bain and Wilkins (2002) have studied real traffic behaviour versus forecasted traffic, concluded that traffic inaccuracy is substantial. Vassallo (2006) has implemented a classification of traffic risk mitigation mechanisms implemented in PPP contracts around the world. Ping Ho (2006) has studied the strategic behaviour of stakeholders in concession PPP contracts. Guasch (2004) has studied the frequency and causes of renegotiation in PPP contracts.

In spite of the great number of research studies dealing with traffic risk in different ways, there is no research study that has approached the influence that the implementation of traffic risk mitigation mechanisms (from now on TRMMs) have on the behaviour of the stakeholders involved in PPP contracts. This strategic behaviour may be analyzed both in the tender and during the life of the contract when the PPP contract has already been awarded.

In this research paper, we intend to study in detail the strategic behaviour of PPP stakeholders depending on the implementation of traffic risk mitigation mechanism. In particular, the aim of this paper is to evaluate whether these mechanisms are useful to reduce opportunistic behaviour with a greater social welfare. We intend to finish this paper with a set of recommendations regarding the implementation of TRMMs.
2.1 Objectives

The objectives of this research paper are:

- To characterize different TRMMs.
- To identify the influence that the implementation of TRMMs will have on the behaviour of the bidders in PPP tenders.
- To identify the stakeholders’ strategic behaviour in PPP contracts depending on whether a TRRM is ultimately implemented or not.
- To evaluate whether the implementation of TRMMs reduces the probability of renegotiation.
- To approach social welfare gains stemming from the implementation of TRMMs.
- To set up some guidelines regarding the implementation of TRMM depending on specific characteristics of PPP projects.

2.2 Hypotheses

This research starts from some hypotheses that we intend to validate theoretically using principal-agent theory and game theory. Subsequently, we intend to contrast the theoretical conclusions obtained with empirical data.

The hypotheses we formulated are the following:

- The implementation of TRMMs prompts bidders to present more realistic offers, particularly regarding traffic forecasts.
- The implementation of traffic risk-mitigation mechanisms reduces renegotiation during the PPP contract length.
- Overall, the implementation of TRMMs is socially more efficient than the full transfer of traffic risk to the PPP contractor.
- If the TRRM implemented is too limited, in such a way that the private promoter has hardly any chance of getting any upside, the attractiveness of this PPP contract for the private sector will be substantially reduced.

2.3 Methodology

The methodology we intend to use includes the following steps:

Phase 1: Approach.

a) Identification and characterization of TRMMs.

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3 For a basic review on game theory see the annex A.
b) Identification of the stakeholders (bidders, government and so on) in PPP projects and definition of their incentives.

c) Analysis of the strategic behaviour of the stakeholders
   - In the tender.
   - During the life of the contract (renegotiation and so on).

d) Identification of the influence of TRMMs in the strategic behaviour of the stakeholders.
   - In the tender.
   - During the life of the contract (renegotiation and so on).

Phase 2: Analysis.

e) Theoretical analysis of the strategic behaviour using principal-agent theory and game theory. Annex A makes a short description of some of the principles of game theory.

f) Empirical contrast of the result based on existing data.

Phase 3: Results.

g) Evaluation of social gains produced by the implementation of TRMMs.

h) Elaboration of a set of policy recommendations regarding the implementation of TRMMs.

As the Figure above shows, the methodology steps can be classified in three phases. The first one intends to identify all the necessary pieces to build a theoretical model that may help prove the formulated hypotheses (section 2.2). The second step theoretically analysis the influence that TRMMs have on the strategic behaviour of bidders in the tender and the ultimate concessionaire during the life of the contract. We intend to empirically contrast the theoretical conclusions as long as enough data is obtained.

2.4 Data Needs

We have already conducted a detailed literature review that provides us with information about TRMMs, traffic risk, and the strategic behaviour of stakeholders in a concession contract. Consequently, the main data source we need to successfully conduct our research are PPP project Databases with the following information for each project:
- Traffic risk mitigation mechanisms implemented, if any.
- Traffic predicted by the winner of the PPP versus real traffic.
- Financial cost of the project.
- Renegotiation of PPP projects in terms of the TRMM implemented.

If the database is not big enough to reach statistical conclusions, we could approach the problem by comparing several case studies in more detail.

We could obtain some information from projects in Spain and Latin America. However, it would be very helpful for us to explore the PPP project database of the European Investment Bank.
Annex A: Essentials of Game Theory

Introduction

Game theory deals with the examination and selection of the “best” options of behaviour (strategy) for each “player” in an interactive situation. The players will choose their moves to maximise their payoff. Each player always assumes that the other player is also trying to maximise their benefits.

The focus of game theory is the interdependence situations in which a group of people is affected by the choices made by every individual within that group. For the interaction to become a strategic game, however, it needs the participants’ mutual awareness of this cross-effect. What the other person does affects you; if you know this, you can react to their actions, or take advance actions to forestall the bad effects their future actions may have on you and to facilitate any good effects, or even take advance actions so as to alter their future reactions to your advantage. The mutual awareness of the cross-effects of actions, and the actions taken as a result of this awareness, constitute the most interesting aspects of strategy. In such an interlinked situation, the interesting questions include:

- What will each individual guess the choices of others?
- What action will each person take? This question is especially interesting when the best action depends on what the others do.
- What is the outcome of these actions? Is this outcome good for the group as a whole?
- Does it make any difference if the group interacts more than once?
- How do answers change if each individual is unsure about the characteristics of others in the group?

There are a lot of circumstances that illustrate interdependence situations in economics, politics, finance, law and even our daily lives.

Game theory has been used in:

- Auctions: both public, as in the case of bonds; and private, for works of art.
- Voting system. According to each voting system (rules), each one will have different strategies.
- Animal conflicts. In areas of scarce fertile females of species and other situations.
• Sustainable use of natural resources. Strategies to keep renewable resources.
• Random drugs testing in sports and workplaces.
• Bankruptcy law. It is expressed in the law which specifies when and how much creditors can collect from a bankrupt company.
• Poison Pill provisions to avoid takeover bids.
• Research & Development expenditure, choosing the best research lines to get the highest profits, for example, in the case of pharmaceutical firms.
• OPEC cartel, determining the oil price and quantity supply.

The main objective of game theory is to analyse these “games” to determine whether there is a “winning strategy or not”. In other words, game theory studies what is the outcome of the strategic interaction among players. The winning strategy, if any, does not necessarily mean that the solution of the game has to be Pareto optimal.

I.- Concepts and Techniques

Some of the key elements of game theory are the following:

• A set of decision-makers interested in the final result of the interaction. Each one of them is referred to as a “player”.
• Nature can be regarded as another player in the game. It does not receive any payoff, and when it plays, the previous situation of the play changes. It can be called player zero.
• Strategic interactions among players. An action conducted by one player at least affects one of the players in the group. One player should assume that the other players are perfect and rational calculators, and they will choose their best response. In other words, one should suppose the best capabilities of their opponents instead of their (worst) limitations.
• Rules to play are the specifications that determine the game.
• Payoffs are the ultimate benefits (or loses) that each player receive. Each player will have a complete numerical scale with which to compare all logically conceivable outcomes of the game, corresponding to each available combination of strategies. Gains or losses for each player are associated with each possible outcome. Game theory assumes that payoffs for one player capture everything in the outcomes of the game that each player cares about.
• The moves. If a game has only one move, each player strategy is just the action taken on that single occasion. But if a game is made up of several moves, then

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4 Poison pills in business are ways to avoid takeover bids by a potential acquirer that attempts to obtain a controlling block of shares in a target company.
the actions of a player who moves later in the game can respond to what other players have done at earlier points.

- A **pure strategy** will be a rule that tells a player what choice to make at each of their possible decision nodes in the game. A **dominant strategy**, for a player in a game, will be this player’s best response to any feasible choice of strategies for the other players.

- **Rationality.** Game theory assumes that players chose their best action to maximise their preferences. Rationality has two essential requirements: complete knowledge of one’s own interests and flawless calculation of what actions will best serve those interests.

  Being rational does not mean sharing the values of other players, or acting according to sensible, ethical or moral principles. It merely means pursuing one’s own value consistently. When one player carries out the analysis of how other players will respond, this player must recognise that the other players calculate the consequences of their choices using their own value or rating system.

- **Equilibrium.** Players are in equilibrium if a change of strategy by any one of them would lead this player to earn less than if they remained with their current strategy. This situation was defined by John Nash and usually called the “Nash Equilibrium.”

  There are games with non-equilibrium solutions. The solution is not stable since at least one of the players can be better off by deviating from the non-equilibrium solution.

- **Pareto Optimum.** It is a particular result in a game whereby none of the players can be in a better situation without making any other individual worse off.

### II.- Representation

There are two ways to represent a game: the extensive form and the normal (or strategic) form.

1.- **Extensive form** (also called game tree)

Game trees are joint decision trees for all the players in a game. They are made up of nodes and branches and are often drawn from left to right across a page (although, they can take any form). The player is specified by a name or number listed by the vertex. Each terminal node has associated with it a set of outcomes for the player involved in the game. The branches illustrate all the possible actions that can be taken by all the players and also indicate the entire possible outcome from the game.
The extensive form can be used to formalize games when the order to play is important. Sequential-move games, like the market-entry game, are easily illustrated using game trees.

2.- Normal form

The normal (or strategic form) game is usually represented by a matrix, which shows the players, strategies, and payoffs (see the example below). More generally it can be represented by any function that associates a payoff for each player with every possible combination of actions. In the example below there are two players; one chooses the row and the other chooses the column. Each player has two strategies, which are specified by the number of rows and the number of columns. In our example, the strategies are to play cooperate or betray. The payoffs are provided in the diagram.

Note that in each cell, the first number represents player No. 1’s payoff and the second one represents player No. 2’s. Suppose that Player 1 plays cooperate and that Player 2 plays betray. Then Player 1 obtains a payoff of -20, and Player 2 obtains 0.
When a game is presented in normal form, it is presumed that each player acts simultaneously or, at least, without knowing the actions of the other. If players have some information about the choices of other players, the game is usually presented in extensive form.

III.- Characteristics

1.- Sequential or simultaneous

In terms of the decision-making timing, there are two types of games: sequential and simultaneous. Sequential games (or dynamic games) are games where the players act sequentially. Consequently, the players have some knowledge about earlier actions. This need not be perfect knowledge about every action of earlier players; it might be very little information. Simultaneous games are the games where the players act simultaneously. This means that each player makes decisions without knowing the decisions made by others. Normal form is used to represent simultaneous games, and extensive form is used to represent sequential ones.

2.- Interest in Conflict

Games are defined as interaction of people’s interests, which usually are opposite; but there are games that could be cooperative. It happens when two or more players do not compete but strain to get the same aim. In this case, it is supposed than the agreement is feasible and it could be better to reach it.

Both cooperative and non-cooperative games have different characteristics. Most economical and social interactions could be expressed like cooperative games. Trade, business, or economic activity offers scope for deals that benefit everyone. Joint ventures can combine the participants’ different skills and generated synergy to produce more than the sum of what they could have produced separately.

3.- Finite or Infinite

A game played once is, in some aspects, simpler, than a game that is played constantly. In the first case, it is not important the impact of actions to the player, because the game will be played only one time; but in the second case, the player will be careful about the consequences of their actions because the other player will be able to take revenge of the first one.

4.- Symmetric and Asymmetric
A symmetric game is a game where the payoffs for playing a particular strategy depend only on the interaction with the other players, and not on who is playing them. If the identities of the players are changed, they would obtain the same payoffs. Many of the commonly studied 2×2 games are symmetric.

5.- Zero Sum and Non-zero Sum

For every combination of strategies in zero-sum games, the total benefit for all players in the game always total zero. In other words, a player gains what the other losses.

Many games studied by game theorists (including prisoner's dilemma in section II) are non-zero-sum games, because some outcomes have net results greater or less than zero. Informally, in non-zero-sum games, a gain by one player does not necessarily correspond with a loss by another.

6.- Type of Information
A game is one of perfect information if all players know the moves previously made by all other players. Thus, only sequential games can be games of perfect information, since in simultaneous games not every player knows the actions of the others.

A game is one of complete information if all factors of the game are common knowledge. Complete information is a term used in economics and game theory to describe an economic situation or game in which knowledge about other market participants or players is available to all participants. Every player knows the payoffs and strategies available to other players. Complete information is one of the theoretical pre-conditions of an efficient perfectly competitive market. If a game is not of complete information, then the individual players would not be able to predict the effect their actions would have on the others players (even if the actor presumed other players would act rationally).

Complete and perfect information are similar but not identical. Complete information refers to a state of knowledge about the structure of the game, while not necessarily having knowledge inside the game. So for example, one may have complete information in the context of a Prisoner's Dilemma, but nonetheless this is a game of imperfect information since one does not know the action of the other player. Despite this distinction, it is useful to remember that any game of incomplete information can be transformed, terminology-wise, into a game of imperfect information. One simply includes nature as a player in the game and conditions payoffs on nature's unknown moves.

IV.- How Does a Game Finish?

How can we determine the behaviour of the players? In sequential-move, the methodology called rollback is often used. It is based on looking ahead and reasoning back. It involves starting to think about what will happen at all the terminal nodes, and literally “rolling back” through the tree to the initial node. It is called backward induction too.

If we consider the prisoners’ dilemma game, it is about two prisoners who have each been offered a deal to turn state's witness (defect) against the other. They can not communicate each other. They had originally agreed to remain in solidarity, (cooperate with each other) i.e. not testify against each other, but since the agreement cannot be enforced, each must choose whether to honour it. If both remain in solidarity, then they will each only be convicted of a minor charge. In this case, they will be punished with 2 years in jail each one. If only one defects, then the authorities will blame the other and let the defector go. In this case, the traitor will be released, but the player who keeps their promise will get 20 years in jail. If they both defect, each will get convicted of a serious charge and get both 10 years in jail. See the graphic forms of the game:
The normal form or the payoff matrix is as follows:

![Payoff Matrix Diagram]

How to solve this game? What strategies are "rational" if both men want to minimize the time they spend in jail? One intuitive way to answer this matter will imagine the rationality of both players: Player 2 might reason as follows: "Two things can happen: Player 1 can confess or Player 1 can keep quiet. Suppose Player 1 confesses. Then I get 20 years if I don't confess, 10 years if I do, so in that case it's best to confess. On the other hand, if Player 1 doesn't confess, and I don't either, I get only 2 years; but in that case, if I confess I can go free. Either way, it's best if I confess. Therefore, I'll confess."

But Player 1 can and presumably will reason in the same way so that they both confess and go to prison for 10 years each. Yet, if they had acted "irrationally," and kept quiet, they each could have gotten off with two years each.

1. Solution using extensive graphics
The best way to know what the result will be is to look at the game tree. The two terminal nodes in a game tree show the outcome for the choices, Cooperate or Betray, for prisoner 2 following what prisoner 1 chose. At node B, if prisoner 2 chooses Cooperate, they achieve their second-best outcome (-2); if they choose Betray, they achieve their best outcome (0). Clearly it is better for prisoner 2 to choose Betray at node b. Rollback tells us to identify Betray as prisoner 2’s optimal action at this node and to identify Cooperate as an action that would never be taken in equilibrium. Similarly, Prisoner 2’s optimal action at node c is Betray. Choosing Betray when prisoner 1 plays Betray gives prisoner 2 their best payoff of –10 from node C, while choosing Cooperate gives them –20.

The best actions for player 2 will be marked with an arrow, at nodes B and C. The other actions are considered pruned from the tree; this means that they are not possible actions for player 2, as shown in the next figure:

To continue analysing this game using rollback, it is necessary to step back to the previous decision node, where Prisoner 1 decides whether they should Cooperate or Betray. Prisoner 1 has to compare the outcome associated with each option in order to decide which choice gives their a better payoff from the game. Because the tree has already been pruned, Prisoner 1 knows that choosing Cooperate will lead to Prisoner 2 choosing Betray and leave Prisoner 1 with a payoff of -20. Choosing Betray leaves Prisoner 1 with a payoff of -10, because Prisoner 2 is sure to choose Betray from node C. The optimal action for Prisoner 1 at node A, then, is to choose Betray. The bottom branch from node A should be marked with an arrow, and the top one should be pruned from the tree. The pruned tree will be:
The remaining branches define the optimal strategies for each player, leading us to the rollback equilibrium outcome of the game. The equilibrium set of payoffs if is circled in the diagram.

2.- Solution using normal graphics

Using normal graphics, we have to check the payoff for each player at the payoff's matrix, and to draw an arrow indicating which payoff if better according to each action.

Analysing the payoff matrix for Player 1, If Player 2 plays Cooperate, then Player 1 prefers play Betray because in that situation they get their best payment. If Player 2 plays Betray, Player 1 will choose Betray, because they get -10 instead of -20. We can check that Player 1 will have a dominant strategy. This means that they will always play Betray despite the Player 2 do (the two blue arrows to point to the same direction).
Analysing the situation for Player 2, we observe that the options are the same (because this is a symmetric game). In this case Player 2’s dominant strategy will be Betray too (see the red arrows).

To arrive at the solution we have to check if the “arrows converge” into some cell. All the cells where the arrows meet will be equilibrium solutions or Nash equilibrium.

Here, the two prisoners are involved in something called a “dominant strategy equilibrium\(^5\)”. In the Prisoners' Dilemma game, to betray is a dominant strategy, (because each one hopes the other keep their promise), only the (betray, betray) option, where both players choose betray, satisfy the requirement of Nash equilibrium. Note that although the (cooperate, cooperate) option is better off for both players compared to the Nash equilibrium; this solution is unstable since either player can obtain extra benefits betraying his/her partner\(^6\).

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\(^5\) Dominant Strategy Equilibrium: If, in a game, each player has a dominant strategy, and each player plays the dominant strategy, then that combination of (dominant) strategies and the corresponding payoffs are said to constitute the dominant strategy equilibrium for that game.

\(^6\) To further details, see *Prisoner's Dilemma* 1993, William Poundstone Anchor book, New York.
The prisoner’s dilemma is called a “simultaneous game” because the players have to decide their testimony simultaneously. This means that each player does not know the other player’s decision before adopting her decision. This game is of complete information, because the payoff of the matrix is known, and the players are assumed to be rational.

The only equilibrium of this game thus leads to the Pareto inefficient outcome. This provides the most famous example that strategic equilibrium typically implies inefficient outcomes, and even can lead to the worst possible outcome (any other outcome is Pareto-dominating the equilibrium outcome.)

This remarkable result—that individually rational action results in both persons being made worse off in terms of their own self-interested purposes—has caused a wide impact in modern social science. The prisoners' dilemma game illustrates the structure of interaction in an oil cartel, or any oligopolistic industry of quantity competition, where each firm has an incentive to ‘spoil’ the market by unilaterally increasing its own output. The same structure of interaction characterizes the problem of providing public goods (free rider problem), i.e. of voluntarily paying taxes.
REFERENCES


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